Unaligned Rebound Attack - Application to KECCAK

Jian Guo

joint with *Alexandre Duc*, *Thomas Peyrin* and *Lei Wei*

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A*STAR Institute for Infocomm Research

Outline

1. Introduction to Hash Functions
   - Introduction
   - Recent Events
   - the SHA-3 Competition
2. The Keccak Sponge function family
   - Compression Function Constructions
   - Keccak
3. Unaligned Rebound Attack
4. Conclusion

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What is hash

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Cryptographic hash functions:

$$\text{arbitrary bit string} \xrightarrow{h} \text{fixed length string}$$

Applications

- digitial signature
- data integrity, checksum
- message authetication code
- random number generators
- digital forensics
- and more ...

Required Properties

- **collision resistance**.
  It should be computationally difficult to
  find x, x' s.t. $x \neq x'$ and $h(x) = h(x')$. — $2^{n/2}$
- **preimage resistance**.
  Given $h(x)$ (not x), find x' s.t. $h(x) = h(x')$. — 2^n
- **second preimage resistance**.
  Given m, find m' s.t. $m \neq m'$ and $h(m) = h(m')$. — 2^{n-k}

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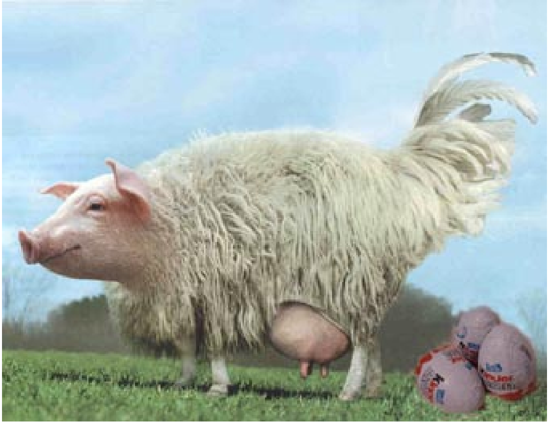
- **pseudo-randomness**.
  $h(key, \cdot)$ should **look like** a random oracle.
- **Unpredictability**.
  predict $h(key, x)$ for unqueried x's
- **Indifferentiability**.
  find "related" sets of input/output values.
- software/hardware **efficiency**
- and more ....

Expectations from Hash Functions



egg producing, wool providing, milk giving, pig

Recent Events in Hash Functions

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- In response, on 3rd March 2006, NIST recommended using `SHA-2` from 2010.
- `SHA-2` follows similar design principle of `SHA-1`, call for `SHA-3` on 2nd Nov 2007.

The SHA−3 Competition

- 2007/11/02: call for submissions.
- 2008/10/31: submission deadline, 64 received.
- 2008/12/09: 51/64 were selected for Round 1.
- 2009/07/24: 14/51 were selected for Round 2.
- 2010/12/10: 5/14 were selected for Round 3.
- 2012/10/03: announcement of winner – KECCAK.

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- **2009/02/25-28: 1st SHA−3 conference, KULeuven.**
- 2009/07/24: 14/51 were selected for Round 2.
- **2010/08/23-24: 2nd SHA−3 conference, UCSB.**
- 2010/12/10: 5/14 were selected for Round 3.
- **2012/03/22-23: 3rd SHA−3 conference, Washington.**
- 2012/10/03: announcement of winner – KECCAK.

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Compression Function Constructions

- block size based, e.g., $H' = E_K(H) \oplus H$, `MD5`, `SHA-1`, `SHA-2`

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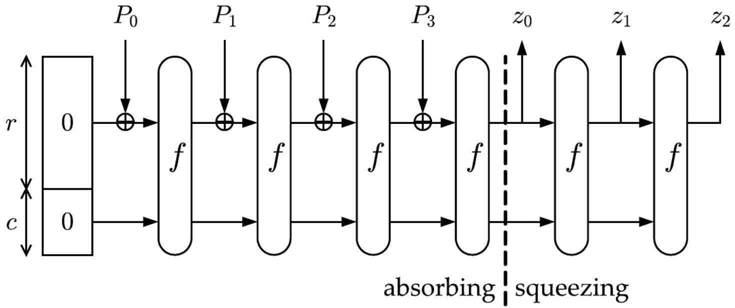
- permutation based, e.g., KECCAK, JH

The Sponge Construction



f: a permutation; r: message rate, i.e., bit size for each message block; c: capacity; z_i: hash outputs;

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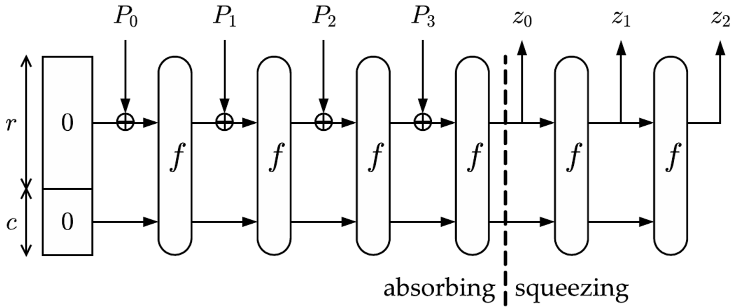


f: a permutation; r: message rate, i.e., bit size for each message block; c: capacity; z_i: hash outputs;

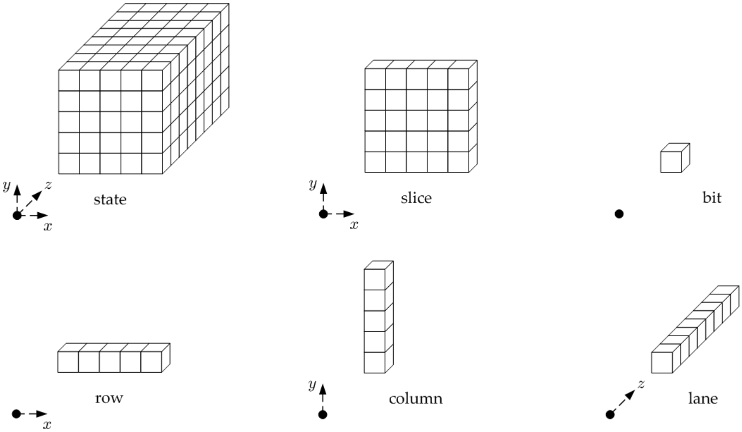
A sponge function is indifferentiable from a random oracle if the permutation is ideal.

The KECCAK Sponge hash function family

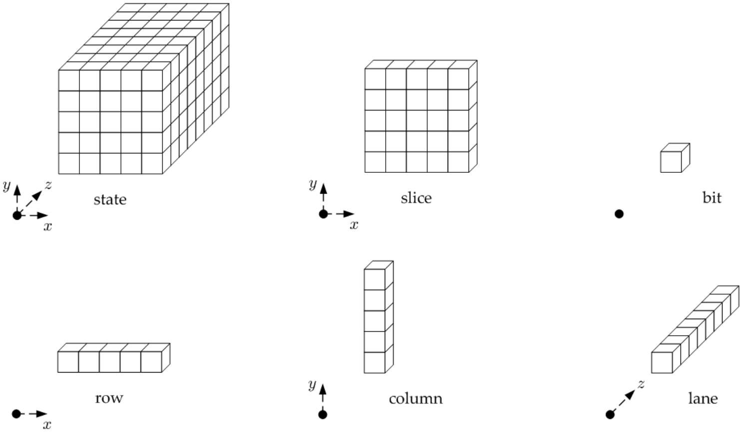
- one 1600-bit permutation
- specified $224, 256, 384, 512$ bit outputs, and ideally supports all output sizes.
- ARX (Addition-Rotation-Xor) design with 5-bit ‘ARX sbox’
- 24 rounds with identical round function up to a difference of constant addition.

The KECCAK Sponge hash function family

The 1600-bit state can be viewed a cube of 5*x*5*x*64.



The 1600-bit state can be viewed a cube of $5x5x64$.



24 rounds, each consists of $\iota \circ \chi \circ \pi \circ \rho \circ \theta$

The KECCAK round function

$$\iota \circ \chi \circ \pi \circ \rho \circ \theta$$

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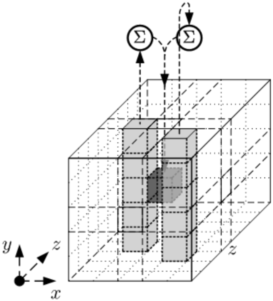

a[x][y][z] = a[x][y][z] + $\sum$ a[x-1][·][z] + $\sum$ a[x+1][·][z-1]
provides inter-slice diffusion, linear

The KECCAK round function

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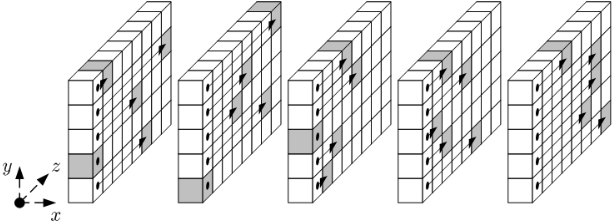

provides intra-lane diffusion, linear, no difference increases for differential path

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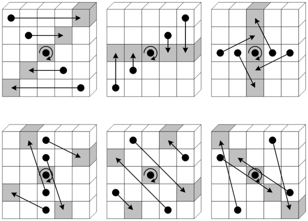

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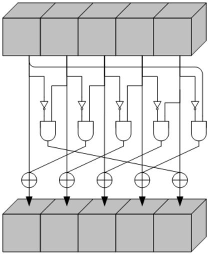

the only non-linear layer, can be viewed as 5-bit sbox

The KECCAK round function

$\boldsymbol{\iota} \circ \chi \circ \pi \circ \rho \circ \theta$

ι: adding a round dependent constant to the first lane, to distinguish each round, and resists slide attack etc. Not essential for differential path

Overall: we can write $\iota \circ \chi \circ (\pi \circ \rho \circ \theta)$ as $\iota \circ \chi \circ \lambda$, and consider only $\chi \circ \lambda$ for differential paths.

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Building High Probability Differential Path

$\theta : a[x][y][z] =$
$a[x][y][z] + \sum_{y=0}^{4} a[x-1][y][z] + \sum_{y=0}^{4} a[x+1][y][z-1]$

Properties: θ propagates the differences slowly, i.e., one bit affects at most 11 other bits; however one bit difference θ^{-1} affects half of the state, around 800 bits.

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Furthermore, if we can keep even number (0, 2, 4) of difference bits in each column, then the $\sum$ terms can be removed, and θ acts like identity. This is called column parity kernel (CPK).

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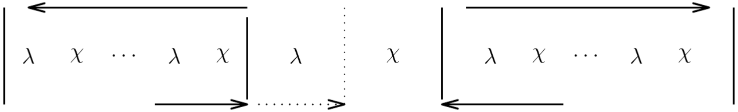
Keep the differential path in the CPK as much as possible! This is possible for at most 3 rounds of KECCAK.

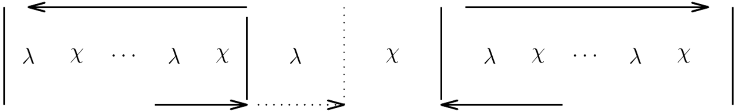
aiming for a differential path with 7 rounds like:
$3R \longrightarrow R \longleftarrow 3R$.

$$\left|\ \lambda \quad \chi \quad \cdots \quad \lambda \quad \chi\ \right| \quad \lambda \quad \vdots \quad \chi \quad \left|\ \lambda \quad \chi \quad \cdots \quad \lambda \quad \chi\ \right|$$

Unaligned Rebound Attack to KECCAK



- differential path for each side is of 3 rounds.



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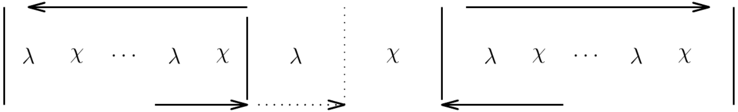

- differential path for each side is of 3 rounds.
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- once a difference match is found, generate all possible solution, and find one confines the differential path in both sides.

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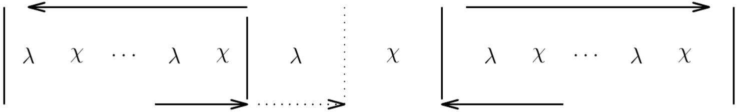

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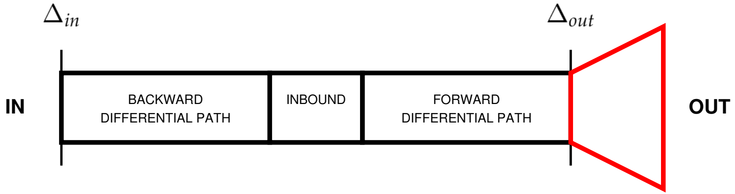
By carefully examine the tradeoff between differential probability and number of differential paths possible, we obtained 7-round differential path with complexity $2^{491.5}$.

Limited Birthday Distinguishers

Given a set of input difference Δ_{in} with size IN and a set of output difference Δ_{out} with size OUT, a bijective function f with b bits, the complexity to find a pair (x, x'), such that $x \oplus x' \in \Delta_{in}$ and $f(x) \oplus f(x') \in \Delta_{out}$, is $\max\{\sqrt{2^b/IN}, \sqrt{2^b/OUT}, 2^b/(IN \cdot OUT)\}$.

If an algorithm finds such a pair faster, we call it a distinguisher.

Application to KECCAK



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- Complexity $2^{491.5}$
- $IN \leq 2^{128.4}$, $OUT \leq 2^{414}$, generic limited birthday attack comes with complexity $2^{1057.6}$.
- hence distinguisher for 8 rounds.

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Attack Comparisons

- J.-P. Aumasson *et al.* (2009):
  zero-sum distinguishers up to 16 rounds of KECCAK-1600 internal permutation with complexity 2^{1024}.
- P. Morawiecki and M. Srebrny (2010):
  preimage attack using SAT solvers, 3 rounds.
- D. Bernstein (2010):
  (second)-preimage attack on 8 rounds with complexity $2^{511.5}$ and 2^{508} bits of memory, using low algebraic degree.
- C. Boura *et al.* (2010-2011):
  zero-sum partitions distinguishers to the full 24-round version of KECCAK-1600 internal permutation with complexity 2^{1590}, using low algebriac degree. Improved by Duan and Lai in 2012 to 2^{1575}.
- I. Dinur (2012): 4-round collision and 5-round near collision for KECCAK-224 and KECCAK-256.
- Ours (2012): 8-round distinguisher with complexity $2^{491.5}$.

End of Talk

Thank you!

Questions?