Provable Security Evaluation of Structures against Impossible Differential Cryptanalysis

Jian Guo
joint with Ruilin Li, Meicheng Liu, Vincent Rijmen, and Bing Sun

NANYANG TECHNOLOGICAL UNIVERSITY

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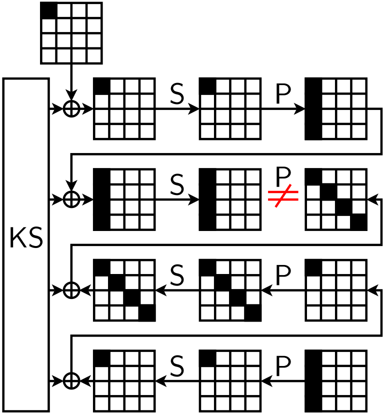
Outline

Motivation

We know how to upper bound the probability of differential / linear characteristics, e.g., 25 active sboxes with prob. $\leq 2^{-6*25} = 2^{-150}$ in **4** consecutive AES rounds,

but less on impossible differential. How to bound the maximum number of round of ID of a given SPN cipher ?

4R AES ID



The structure

SP Network:

$$AK \circ S \circ P \circ \underbrace{AK \circ S \circ P}_{\text{1 round}} \circ \cdots \circ AK \circ S \circ P \circ AK \circ S \circ P$$

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Assume each Sbox can take all permutations S' of the same size,

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The Structure:

$$S' \circ P \circ \underbrace{S' \circ P}_{1 \text{ round}} \circ \cdots \circ S' \circ P \circ S'$$

Some Properties I

Since S' can take any permutation, if the differential $\alpha \xrightarrow{S'} \beta$ is possible, then the differential $\alpha' \xrightarrow{S'} \beta'$ is also possible for all (α', β') sharing the same truncated characteristic with (α, β). Hence such property preserves for $S' \circ P$, and for $(S' \circ P)^r \circ S'$ for any $r \geq 0$.

Some Properties II

If the differentials $\alpha_1 \xrightarrow{S' \circ P \circ S'} \beta_1$ and $\alpha_2 \xrightarrow{S' \circ P \circ S'} \beta_2$ are possible, then $\alpha_1 | \alpha_2 \xrightarrow{S' \circ P \circ S'} \beta_1 | \beta_2$ is also possible. Hence such property preserves for any $(S' \circ P)^r \circ S'$ for any $r \geq 0$.

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The Contrapositive:

If $\alpha \xrightarrow{(S' \circ P)^r \circ S'} \beta$ is impossible, then $\alpha' \xrightarrow{(S' \circ P)^r \circ S'} \beta'$ is impossible for some α' and β' with single active nibble.

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Useful Induction:

Then, the search of impossible differential of $r + 1$ rounds is reduced to checking all m^2 (v.s. previous 2^{2m}) such (α', β') pairs (m denotes the number of nibbles of the state).

How to determine the maximum round of ID

Represent the state as a vector, and P as a matrix, denote the truncated characteristic matrix as P^*, determine minimum r_1 such that $(P^*)^{r_1}$ is all one matrix, similarly minimum r_2 such that $(P^*)^{-r_2}$ is all one matrix, then the max round of ID is $r_1 + r_2$.

Determine the maximum round of ID - example of AES

The AES MixColumn Matrix

$$\begin{pmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{pmatrix} \xrightarrow{\text{truncated characteristic}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Determine the maximum round of ID - example of AES

$$P^* = \begin{pmatrix}
1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1\\
1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1\\
1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1\\
1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1\\
0&0&0&1&1&0&0&0&0&1&0&0&0&0&1&0\\
0&0&0&1&1&0&0&0&0&1&0&0&0&0&1&0\\
0&0&0&1&1&0&0&0&0&1&0&0&0&0&1&0\\
0&0&0&1&1&0&0&0&0&1&0&0&0&0&1&0\\
0&0&1&0&0&0&0&1&1&0&0&0&0&1&0&0\\
0&0&1&0&0&0&0&1&1&0&0&0&0&1&0&0\\
0&0&1&0&0&0&0&1&1&0&0&0&0&1&0&0\\
0&0&1&0&0&0&0&1&1&0&0&0&0&1&0&0\\
0&1&0&0&0&0&1&0&0&0&0&1&1&0&0&0\\
0&1&0&0&0&0&1&0&0&0&0&1&1&0&0&0\\
0&1&0&0&0&0&1&0&0&0&0&1&1&0&0&0\\
0&1&0&0&0&0&1&0&0&0&0&1&1&0&0&0
\end{pmatrix}$$

and $(P^*)^2 = 1$, hence $r_1 = 2$ and similarly $r_2 = 2$, maximum round is $r_1 + r_2 = 4$.

Results

- Proved, without considering the details of Sboxes, ID of AES is bounded by 4 rounds, and 8 rounds for Camellia w/o FL. In other words, the only way to find longer ID is to consider the Sbox properties.
- Gave simple way to determine such bounds.
- Due to the duality of ID cryptanalysis and zero-correlation cryptanalysis, similar results apply to ZC as well.

EoT

Thank you!

Questions?